Statistics

Lecture 12

Feb 19-8:47 AM

SG 21

I Surveyed 275 Students and 16% of them were STEM majors. $n=275$, $\hat{P}=.16$

$x = n\hat{P} = 275(.16) = 44$

if decimal → Round-up

Find 90% Conf. interval for the proportion of all Students that are STEM majors.

C-level = .9

STAT | TESTS | 1-PropZInt

$x = 44$

$n = 275$

C-level = .9

Calculate

$E = \frac{.20 - .12}{2} = .04$

$\hat{P} = \frac{.20 + .12}{2} = .16$

$.12 < p < .20$

Since $\hat{P}$ was in 2-decimal, we round to 2-dec.

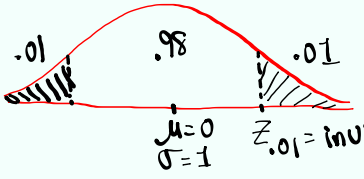
we are 90% Confident that between 12% & 20% of all students are STEM majors.

Nov 17-5:08 PM

Find min. sample size needed if we wish to construct 98% Conf. interval for the prop. of all students that are STEM majors and error not to exceed 5%.

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.16)(.84)\left(\frac{2.326}{.05}\right)^2$$

$$= 290.856 \approx \boxed{291}$$



$z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

Suppose $\hat{p}$ & $\hat{q}$ were unknown — use .5 for each.

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{2.326}{.05}\right)^2 = 541.0276 \approx \boxed{542}$$

Nov 17-5:17 PM

SG 22

Estimating Population Mean μ:

Answer: $\ldots < \mu < \ldots$

Format: $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample Mean, Point-Estimate

E: Margin of error

Case I: σ Known

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

C.V. $z_{\alpha/2}$ → invNorm

Conf. Interval → ZInterval

inpt: Stats

Nov 17-5:26 PM

Given $n=32$, $\bar{x}=86$, $\sigma=10$

Find 99% Conf. interval for Pop. mean.

Since σ is Known $\quad < \mu <$

STAT → TESTS → ZInterval

inpt: STATS

$\sigma = 10$

$\bar{x} = 86$ (whole)

$n = 32$

C-level: .99

Calculate

→ Round to whole #.

$81 < \mu < 91$

$E = \frac{91-81}{2} = 5$

$\bar{x} = \frac{91+81}{2} = 86$

Nov 17-5:31 PM

I randomly selected 35 students, their mean age was 31.5 yrs. $n=35$, $\bar{x}=31.5$

No C-level → .95

Find Conf. interval for the mean age of all students assuming the standard dev. of ages of all students is 7.6 yrs.

$\sigma = 7.6$

$< \mu <$

σ known → Z Interval

inpt: STATS

$\sigma = 7.6$

$\bar{x} = 31.5$ (1-dec.)

$n = 35$

C-level: .95

Calculate

→ we round to 1 dec.

$29.0 < \mu < 34.0$

we are 95% confident that the mean age of all students is between 29 & 34 yrs.

$E = \frac{34-29}{2} = 2.5$

$\bar{x} = \frac{34+29}{2} = 31.5$

Nov 17-5:36 PM

Estimating Population Mean μ: (SG 22)

Answer: $\ldots < \mu < \ldots$

Format: $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample Mean, Point-Estimate

E: Margin of error

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
	$df = n - 1$
C.V. $z_{\alpha/2}$ → invNorm	C.V. $t_{\alpha/2}$ → invT
Conf. Interval → ZInterval	Conf. Interval → TInterval
inpt: Stats	inpt: STATS

Nov 17-5:26 PM

Given $n = 12$ $\bar{x} = 84.6$ $s = 9.5$

C-level : .9

Find 90% Conf. interval for pop. mean μ.

σ unknown → TInterval $< \mu <$

inpt: STATS

$E = \frac{89.5 - 79.7}{2} = 4.9$

$\bar{x} = \frac{89.5 + 79.7}{2} = 84.6$

$\bar{x} = 84.6$ (1-dec) Round to 1-dec.

$s = 9.5$

$n = 12 \rightarrow df = n - 1 = 11$

C-level : .9

Calculate

$79.7 < \mu < 89.5$

Nov 17-5:50 PM

I randomly Selected 15 nurses, their mean monthly Salary was $6750 with Standard deviation of $350.

$n=15$
$\bar{x}=6750$
$S=350$

No C-level → .95

Find Conf. interval for the mean Salary of all nurses.

$<\mu<$

σ unknown → TInterval

inpt: STATS
$\bar{x}=6750$ (whole) → Round to whole #.
$S=350$
$n=15$
C-level: .95
Calculate

$df=n-1=14$

$6556<\mu<6944$

$E=\frac{6944-6556}{2}=194$

$\bar{x}=\frac{6944+6556}{2}=6750$

Nov 17-5:58 PM

I randomly Selected 10 Students. Here are their ages.

Store in a list, Find $\bar{x}$ & S rounded to 1-dec.

26	20	18	20	19
32	35	25	40	36

$\bar{x}=27.1$
$S=8.1$

Find 99% Conf. interval for the mean age of students.

σ unknown → TInterval

inpt: STATS
$\bar{x}=27.1$ (1-dec.)
$S=8.1$
$n=10$
C-level: .99
Calculate

Round to 1-dec.

$18.8<\mu<35.4$

$E=\frac{35.4-18.8}{2}=8.3$

$\bar{x}=\frac{35.4+18.8}{2}=27.1$

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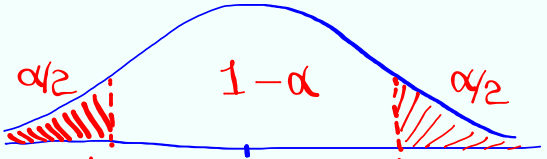
t-dist:

1) Symmetric, Bell-Shape, total Area = 1

2) $\mu = 0$, σ unknown

3) It comes with degrees of freedom
df



How to find $t_{\alpha/2}$:

2nd VARS

invT(LeftArea, df)

Nov 17-6:16 PM

Find $t_{.02}$ with df = 14.

$t_{.02}$ → Area on the right-Tail

$\alpha/2 = .02 \rightarrow \alpha = .04$ Sig. level

$1 - \alpha = .96$ Middle Area

$.96 = 96\%$ C-level

$t_{.02} = \text{invT}(.98, 14) = 2.264$

-2.264 2.264

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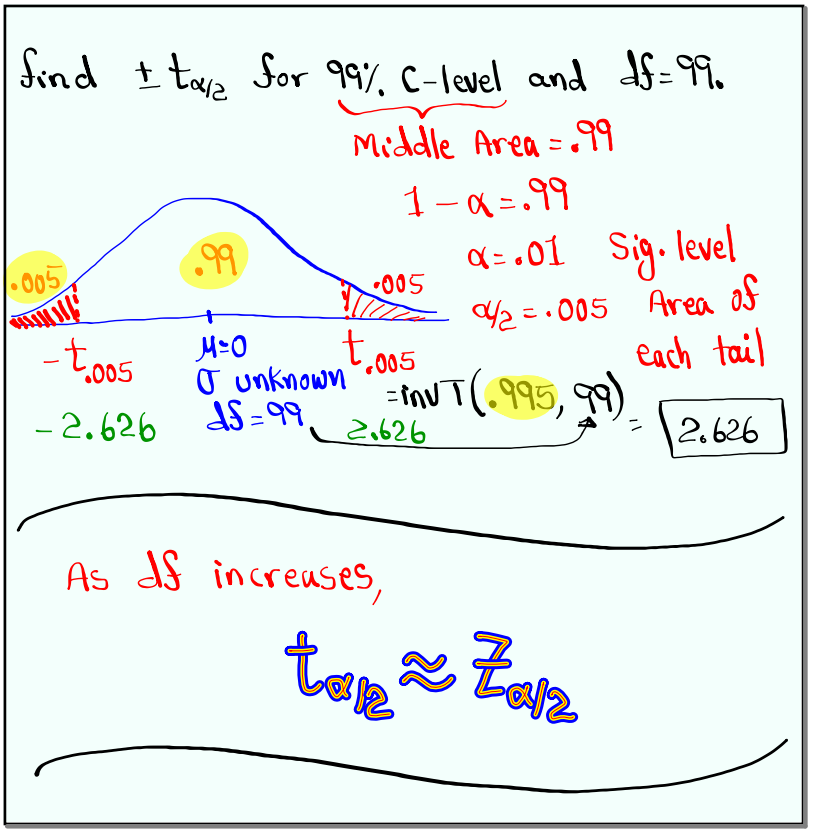

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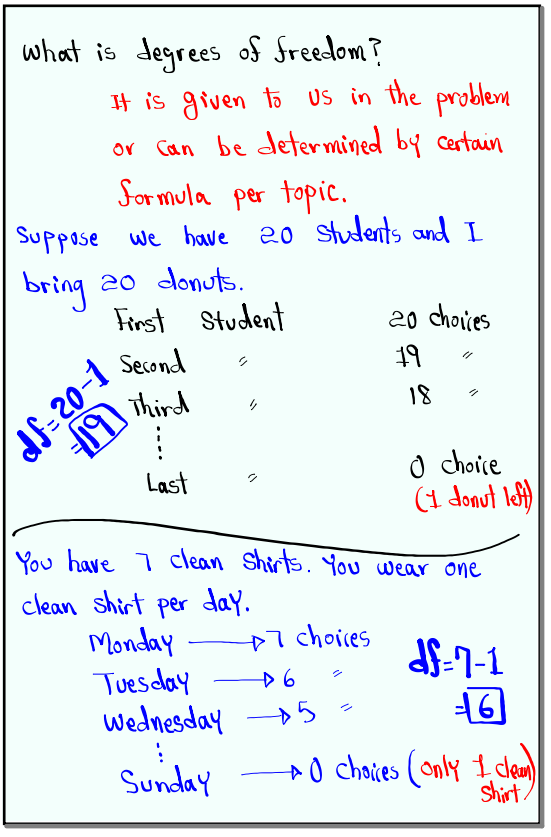

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How to determine min. Sample Size to Construct Conf. interval for μ:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

with Some algebra $\rightarrow$ $$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

if decimal $\rightarrow$ Round-up

If σ is unknown $\rightarrow$ use S in place of σ

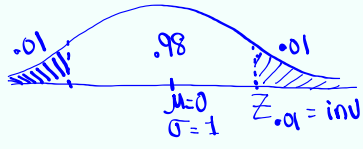
$$n = \left(\frac{z_{\alpha/2} \cdot S}{E}\right)^2$$

Nov 17-6:37 PM

Find min. Sample Size needed to Construct 98% Conf. interval for pop. mean if $\sigma = 18$ & $E = 5$.

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{2.326 \cdot 18}{5}\right)^2 = 70.117\ldots$$

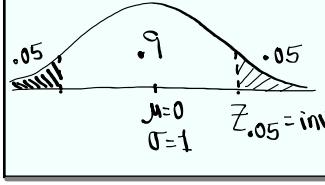
$$\approx \boxed{71}$$



Redo with C-level 90% and $E = 8$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 = \left(\frac{1.645 \cdot 18}{8}\right)^2 = 13.699\ldots$$

$$\approx \boxed{14}$$



Nov 17-6:43 PM

Given $S=7.5$, $E=10$

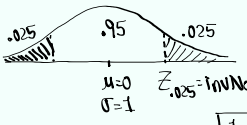
Find min. Sample Size needed to Construct no C-level → 95% Conf. interval for pop. mean μ.

σ is unknown → $n=\left(\frac{z_{\alpha/2}\cdot S}{E}\right)^2$

$=\left(\frac{1.960\cdot 7.5}{10}\right)^2$

$=2.1609$

$n\approx 3$



$z_{.025}=\text{invNorm}(.975,0,1)$

$=1.960$

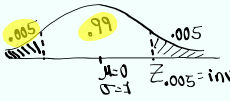
Redo with $E=5$

$n=\left(\frac{z_{\alpha/2}\cdot S}{E}\right)^2=\left(\frac{1.960\cdot 7.5}{5}\right)^2=8.6436$ $n\approx 9$

Redo with 99% C-level & $E=5$

$n=\left(\frac{z_{\alpha/2}\cdot S}{E}\right)^2=\left(\frac{2.576\cdot 7.5}{5}\right)^2=14.93\ldots$

≈ 15



$z_{.005}=\text{invNorm}(.995,0,1)=2.576$

SG 22

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